7^{th} Exercise in Digital Information Processing

1. Matrix potention with eigen-values

   - Given is a system of order 2 with equation:

   $$y(n) = a_1 u(n-1) + a_0 u(n-2) - b_1 y(n-1) - b_0 y(n-2) \tag{1}$$

   - Sketch an equivalent block-diagram.
   - Define two state variables $x_1(n)$ and $x_2(n)$ and rewrite the system equation for $x_1(n+1)$ and $x_2(n+1)$
   - Solve the system with matrix potention for the variables:
     $b_0 = -3/4$ and $b_1 = 1$

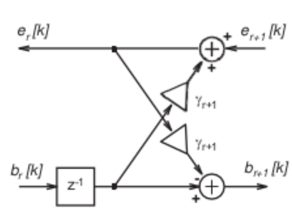
2. Stochastic signals and model system

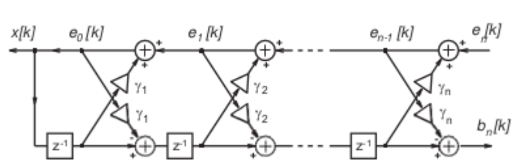
   - Given is a system of order 2 with the equation:

   $$x[n] = q[n] - a_1 x[n-1] + a_2 x[n-2] \tag{2}$$

   All parameters at $t = 0$ are zero.
   The system is excited with the delta function $q[n] = \delta[n]$.
   Furthermore, the values of the autocorrelation are given:
   $S_{xx}[0] = \frac{12}{5}, \quad S_{xx}[1] = -\frac{8}{5}, \quad S_{xx}[2] = \frac{26}{15}.$

   - Knowing the exication, the model and the values of the autocorrelation, which approaches can be used to estimate a_1 and a_2? Use them to estimate a_1 and a_2.
   - Compute $x[0]$ to $x[3]$ using the system $x[n]$ and the calculated model parameters a_1 and a_2.





   - Use a inverse Lattice-structure as given depicted above to test, if this inverse Filter will produce the correct values for $x[2]$.

- Having the model parameters, compute $x[0]$ to $x[3]$ and estimate the autocorrelation values using the consistent autocorrelation estimator

$$\hat{S}_{XX}[|\kappa|] = \frac{1}{N} \sum_{\kappa=0}^{N-1-|\kappa|} x[n]x[n+\kappa] \tag{3}$$

  and the first 4 elements of $x[n]$ to compute $\hat{S}_{XX}[0]$ to $\hat{S}_{XX}[2]$.
  Compare the results to the true values of the autocorrelation function.

- Which equation was used to calculate the corrent values of the autocorrelation?